Partonic Interpretation of Generalized Parton Distributions

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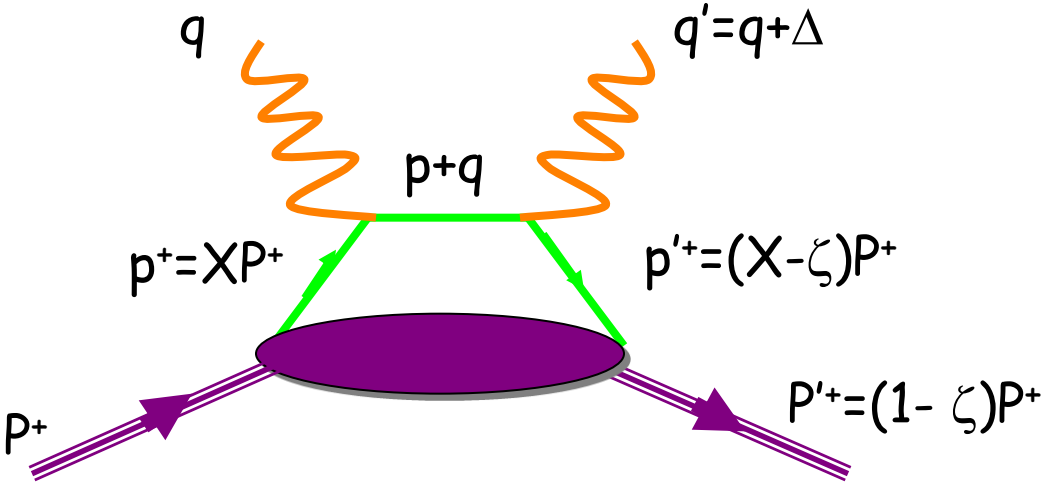
Outline

✓ A new interpretation of the ERBL region based on the analytic properties of GPDs
G. Goldstein and S.L., hep-ph 0610…(2010)

✓ Role and limitations of Dispersion Relations in GPD analyses
G. Goldstein and S.L., Phys. Rev.D (2009)

✓ Connection between GPDs and TMDs
S.L. and S.K. Taneja, Phys. Rev.D (2005) and in preparation

Off forward Parton Distributions (GPDs) are embedded in soft matrix elements for deeply virtual Compton scattering (DVCS)



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are needed to see this picture.

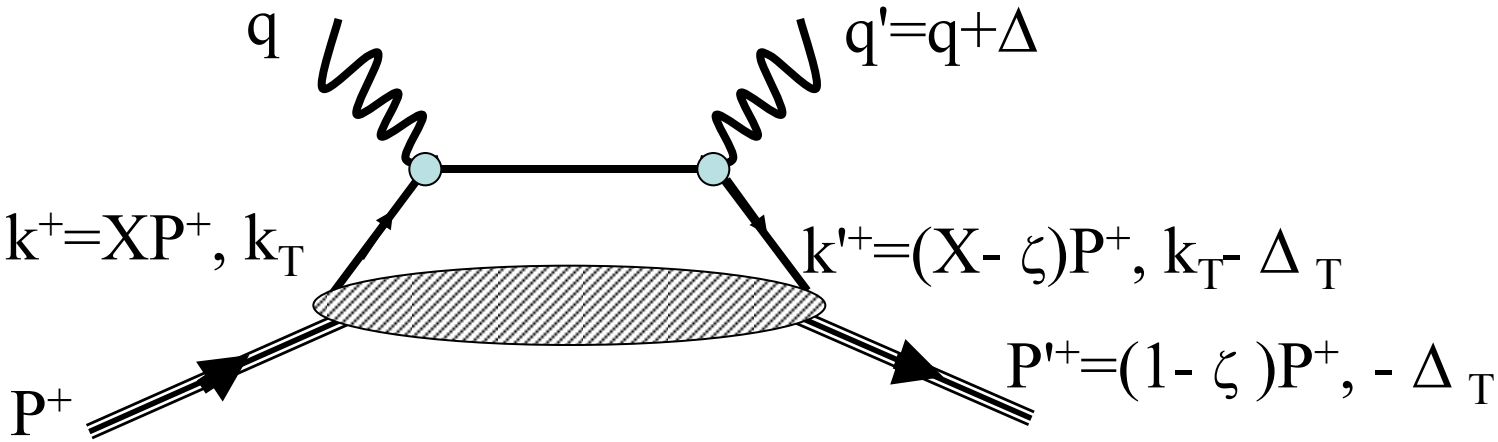
$$\frac{1}{(p+q)^2 - m^2 + i\varepsilon} = PV\frac{1}{(p+q)^2 - m^2} - i\pi\delta((p+q)^2 - m^2)$$

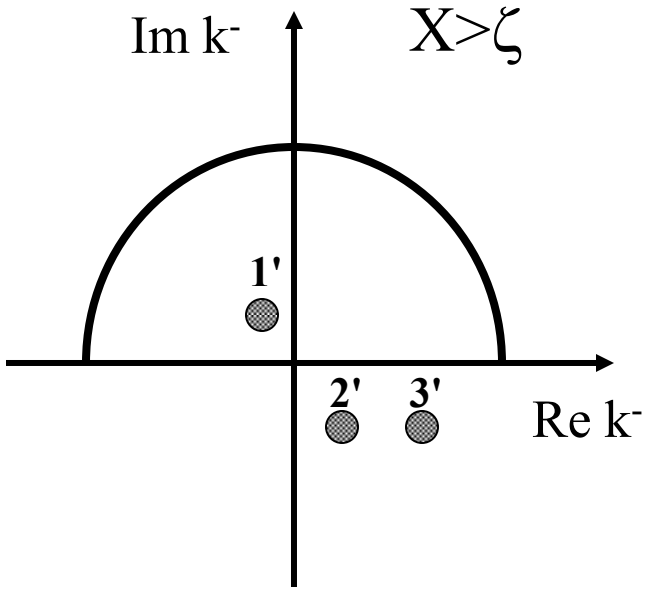
$$\rightarrow \frac{1}{-Q^2 + 2(pq) + i\varepsilon} \rightarrow \frac{1}{-Q^2/2(Pq) + (pq)/(Pq)} = \frac{1}{-\zeta + X}$$

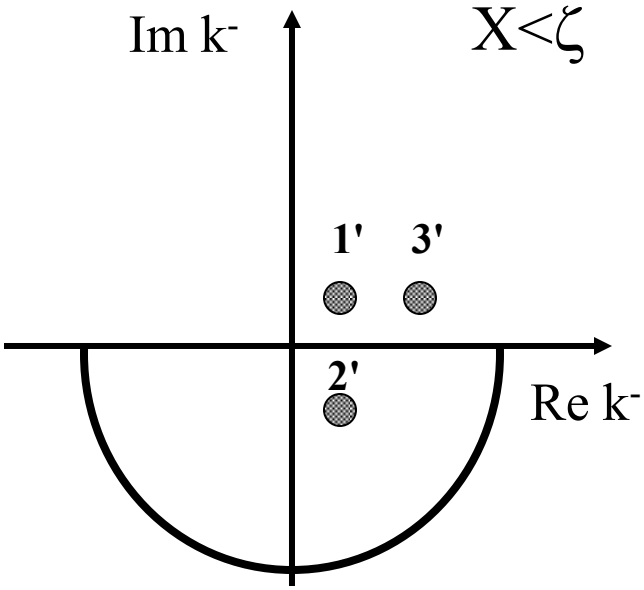
Amplitude

$$F_q = P.V.\int_{-1+\zeta} dX F_q(X,\zeta,t)\left[\frac{1}{\zeta - X} - \frac{1}{X}\right] + i\,\pi e_q^2 F_q(\zeta,\zeta,t)$$

DVCS Kinematics

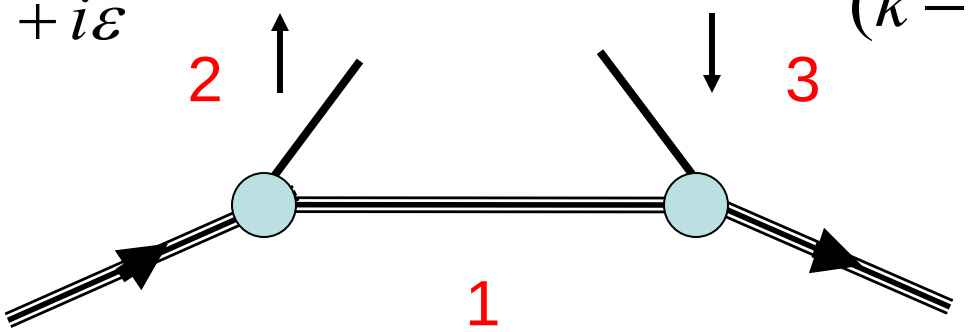






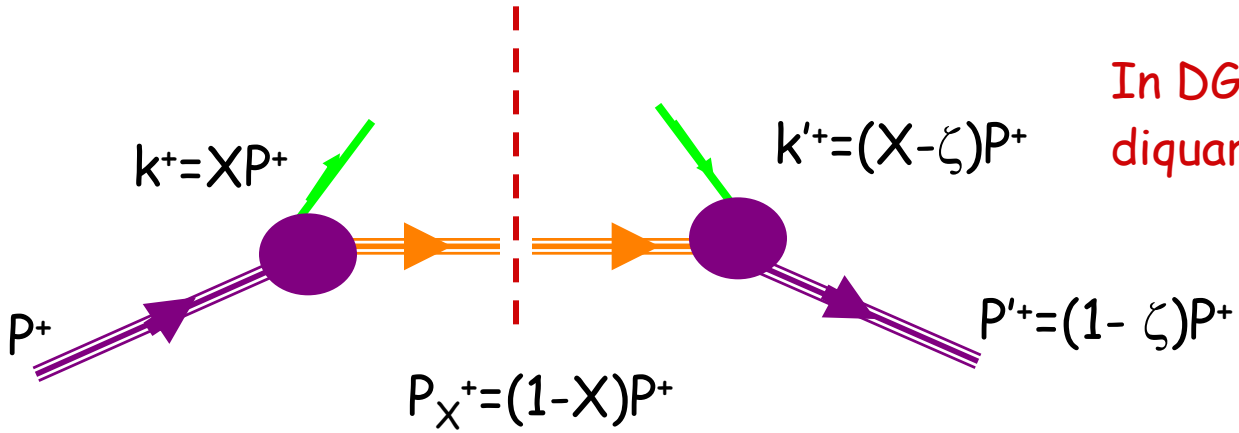
$$k^2 - m^2 + i\varepsilon$$

$$(k-\Delta)^2 - m^2 + i\varepsilon$$



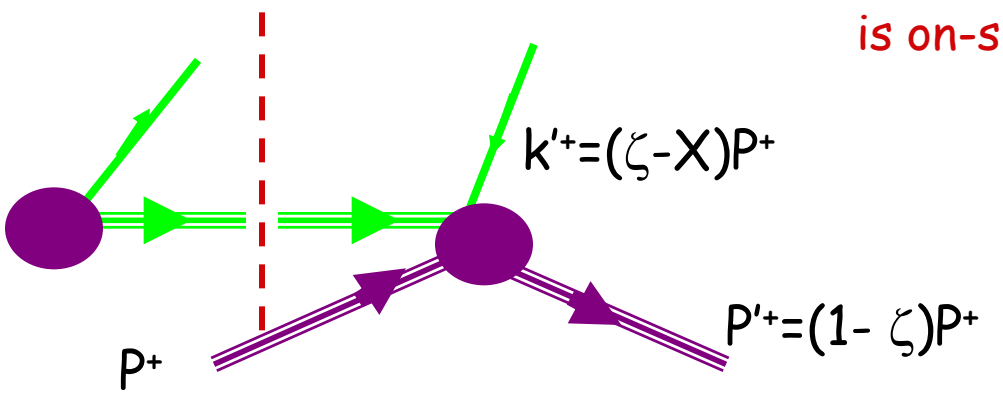
$$(P-k)^2 - M_X{}^2 + i\varepsilon$$

$$H(X,\zeta,t) = \sum_n \langle P'|\psi^+|n\rangle\langle n|\psi|P\rangle\, \delta\left[(X-\zeta)P^+ + p_n^+ - P'^+\right]$$



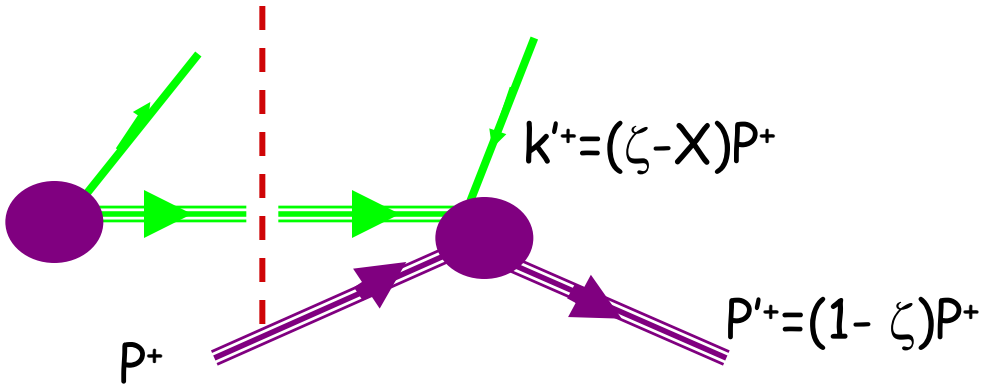
In DGLAP region spectator with diquark q. numbers is on-shell

In ERBL region struck quark, k, is on-shell



Analysis done for DIS/forward case by Jaffe NPB(1983)

ERBL region corresponds to semi-disconnected diagrams:
no partonic interpretation



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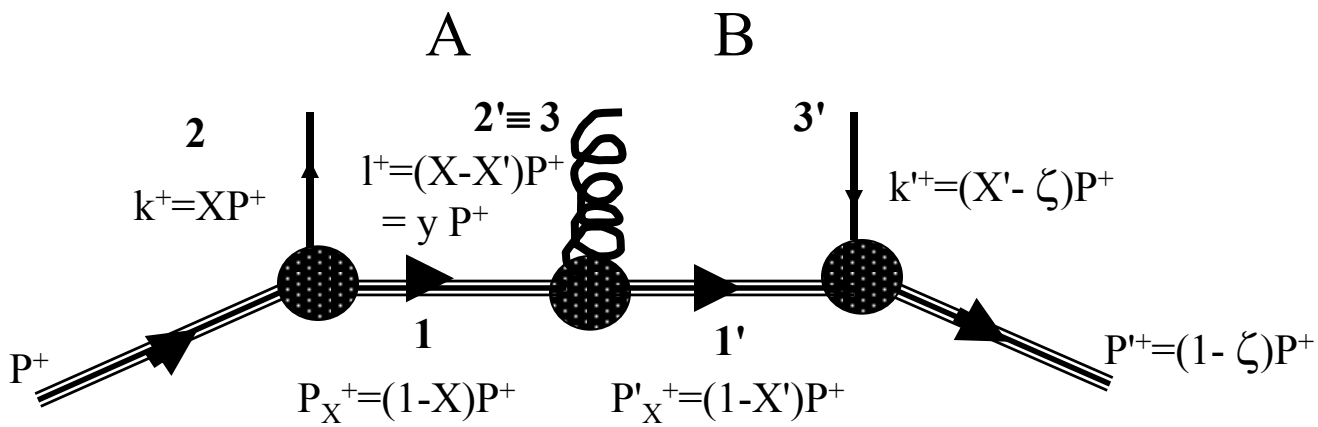
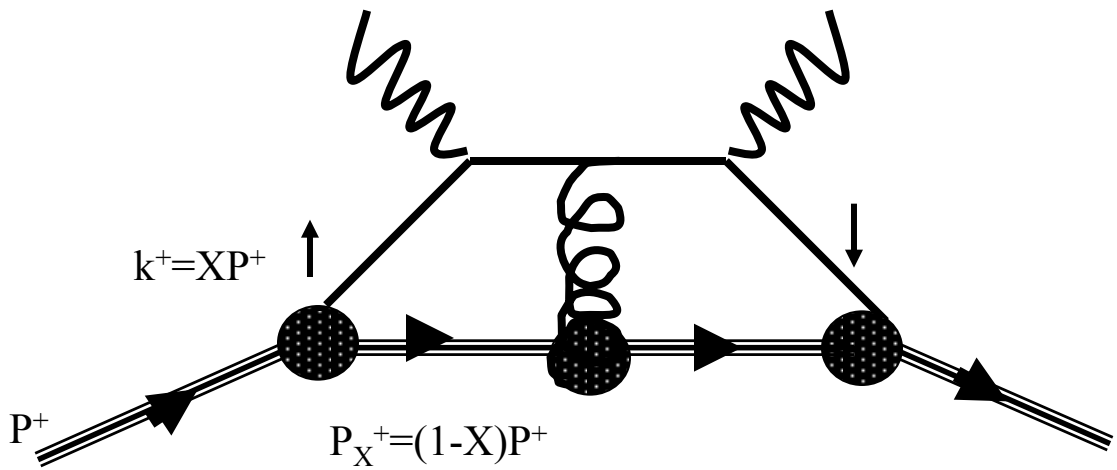
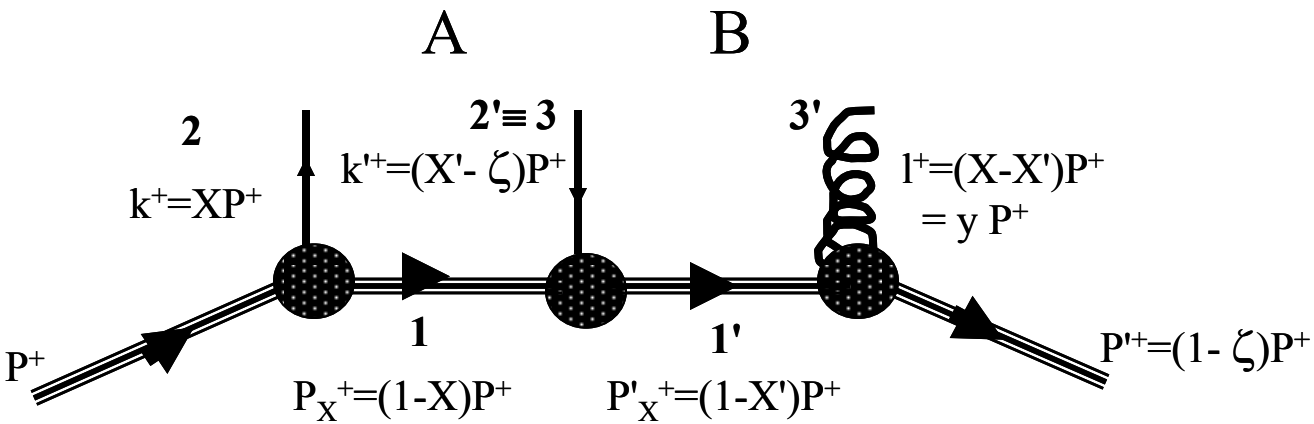
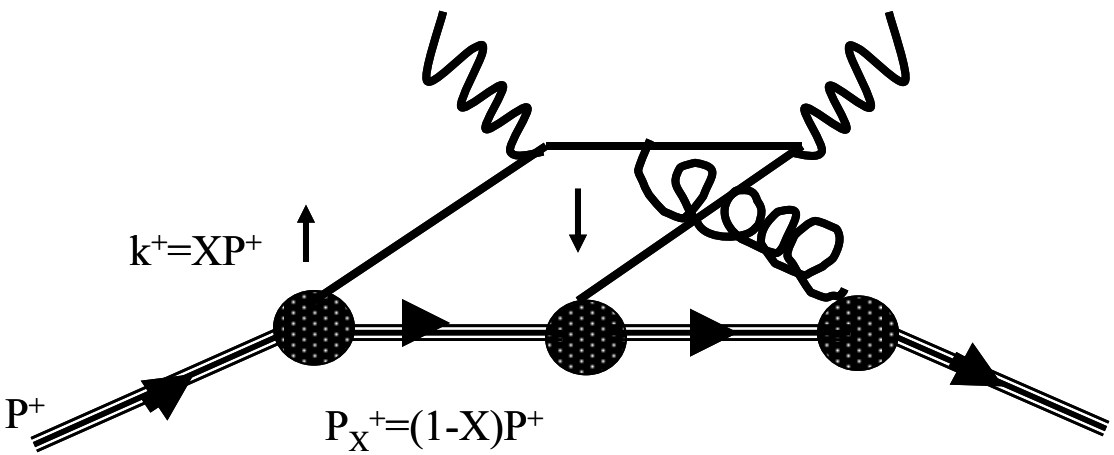
In order to give a partonic interpretation we consider multiparton configurations ⇒ FSI





Planar





Non-Planar

Summary of part 1: GPDs in ERBL region can be described within QCD, consistently with factorization theorems,
<u>only</u> by multiparton configurations

Dispersion Relations

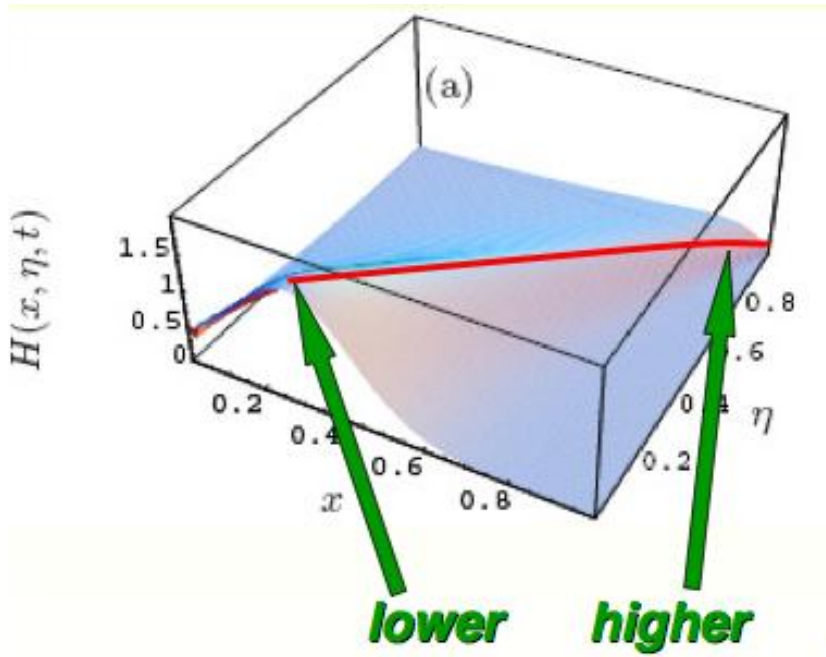
G.Goldstein and S.L.,PRD'09,arXiv:0905.4753 [hep-ph]

(Anikin, Teryaev, Diehl, Ivanov, Vanderhaeghen…)

$$\mathrm{Re}T(\nu,t,Q^2) = \frac{2\nu}{\pi}\int_{\nu_{threshold}}^{\infty} d\nu' \frac{\mathrm{Im}T(\nu',t,Q^2)}{\nu'^2-\nu^2},$$

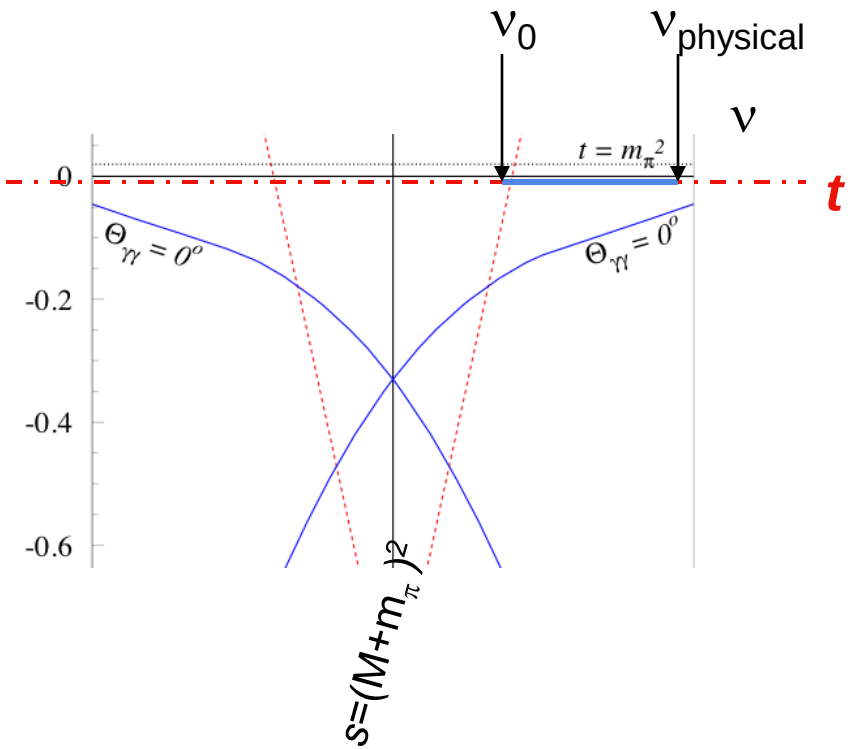
$$\Downarrow \qquad \upsilon = \frac{Q^2}{2\xi}$$

$$\Im m(\mathcal{H}_f(\xi,t)) = H(\xi,\xi,t) \text{ and } \Re e(\mathcal{H}_f(\xi,t)) = \frac{1}{\pi} P.V. \int_{-1}^{+1} dx \frac{H(x,x,t)}{x-\xi}. + C(t)$$



Where is threshold?

Viewed this way a quark + spectator cannot be on their mass shell but hadronic jets must have some threshold. This threshold ("physical threshold") is much higher than what required for the dispersion relations to be valid



$\nu_{phys} \neq \nu_o$ ifmasses in the two-body scattering problem are different!

- Continuum starts at $s = (M+m_\pi)^2 \Rightarrow$ lowest hadronic threshold.
- How to fill the gap? Analytic continuation?

Dispersion relations cannot be directly applied to DVCS because one misses a fundamental hypothesis: "all intermediate states need to be summed over"

This happens because "t" is not zero $\Rightarrow$ t-dependent threshold cuts out physical states

It is not an issue in DIS (see your favorite textbook, LeBellac, Muta, Jaffe's lectures…) because of optical theorem

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are needed to see this picture.

From DR

to Mellin moments expansion

Dispersion Relations (brief parenthesis…)

Dispersion

Direct

Difference

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TIFF (LZW) decompressor
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QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Direct

↕

Dispersion

DVCS

One proceeds backwards, from polynomiality $\Rightarrow$ analytic properties (Teryaev)

However, for DVCS one is forced to look into the nature of intermediate states because there is no optical theorem

t-dependent thresholds are important: counter-intuitively as Q^2 increases the DRs start failing because the physical threshold is farther away from the continuum one (from factorization)

Is the mismatch between the limits obtained from factorization and the physical limits from DRs a signature of the "limits of standard kinematical approximations"? (Collins, Rogers, Stasto and Accardi, Qiu)

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When deeply virtual processes involve directly final states
- like in exclusive or semi-inclusive processes - "standard kinematic approximations should be questioned"
(Collins, Rogers, Stasto, 2007)

(we write ζ but it is equivalent in ξ), so that

$$H(X,\zeta,t) \to \int dm_J^2 \rho(m_J^2) H(\zeta, \left(1+\frac{m_J^2}{Q^2}\right), \zeta, t)$$

$$H(\xi',\xi',t)/(\xi-\xi')$$

is not the same as

$$\Im m A(\nu',t)/(\nu-\nu')$$

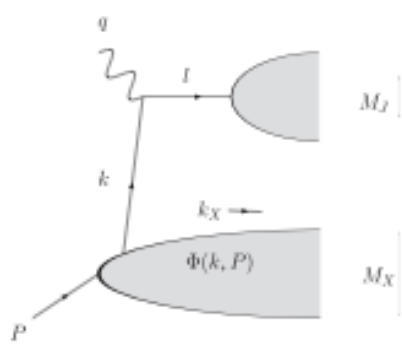

FIG. 2. The amplitude for $\gamma^* p$ scattering into two jets with fixed masses.

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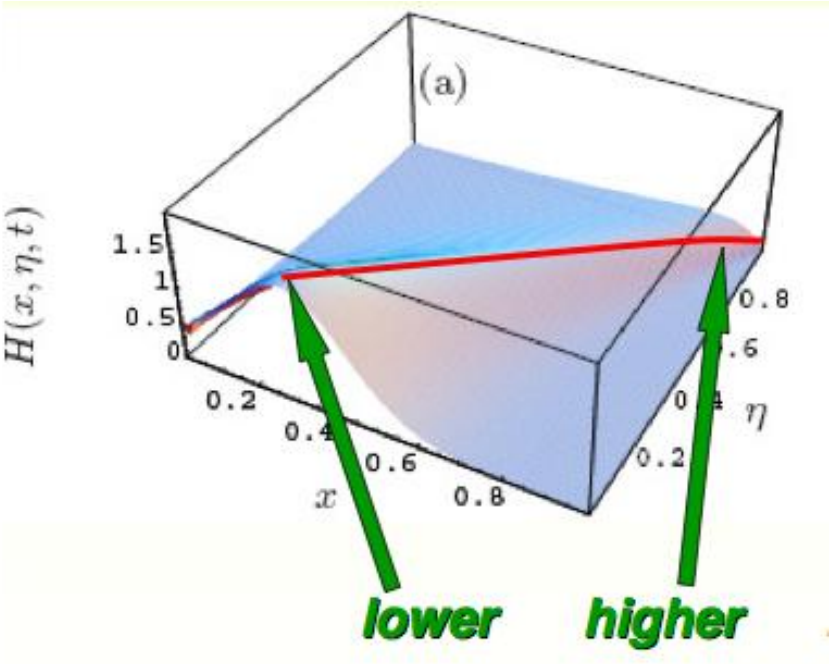
Summary of part 2: dispersion relations cannot be applied straighforwardly to DVCS.

The "ridge" does not seem to contain all the information



On the connection between TMDs and GPDs

Liuti and Taneja, PRD (2004) + in preparation

$$\nu W_2(x) = \sum_i e_i^2 x \int d^2\mathbf{k}\ f(x,\mathbf{k}) \tag{11a}$$

$$F(\mathbf{\Delta}) = \sum_i e_i \int d^2\mathbf{k} \int_0^1 dx\, f(x,\mathbf{k},\mathbf{k}+(1-x)\mathbf{\Delta}), \tag{11b}$$

where:

$$f(x,\mathbf{k},\mathbf{k}') = \phi^*(x,\mathbf{k})\phi(x,\mathbf{k}'), \tag{12}$$

with $\mathbf{k}' \equiv \mathbf{k} + (1-x)\mathbf{\Delta}$, is a non-diagonal intrinsic momentum distribution. The diagonal term can be written as:

$$f(x,\mathbf{k}) = \mid \phi(x,\mathbf{k}) \mid^2 . \tag{13}$$

By comparing Eqs.(11a, 11b), with Eqs.(3a, 3b), we find that the relation between the transverse momentum and the transverse separation of quarks inside a hadron is obtained through a non-diagonal distribution in transverse coordinate space, $q(x,\mathbf{b},\mathbf{b}')$:

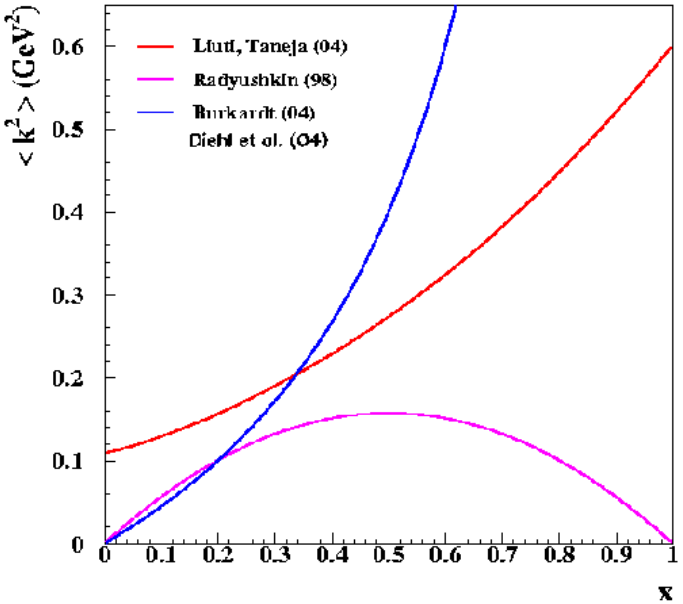
$$f(x,\mathbf{k}) = \int d^2\mathbf{b} \int d^2\mathbf{b}'\, e^{i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\, q(x,\mathbf{b},\mathbf{b}'), \tag{14}$$

where we define:

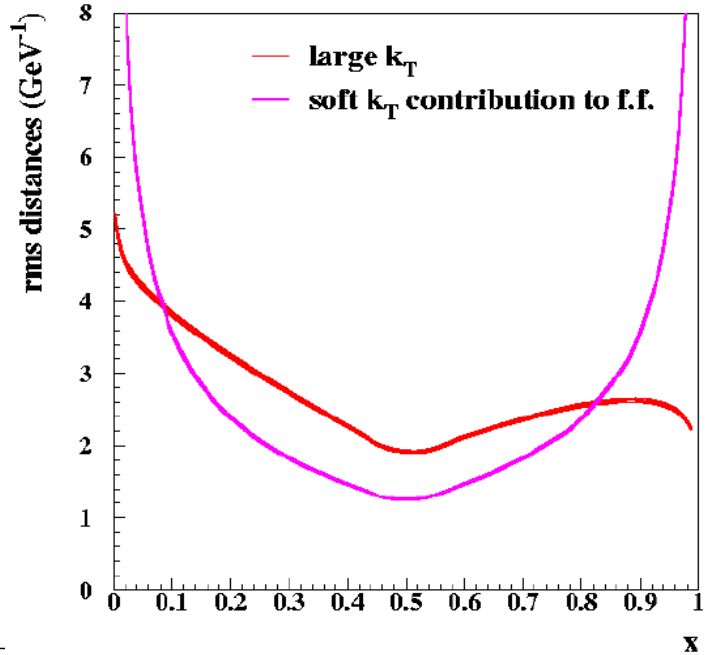
$$q(x,\mathbf{b},\mathbf{b}') = \Psi^*(x,\mathbf{b}')\Psi(x,\mathbf{b}), \tag{15a}$$

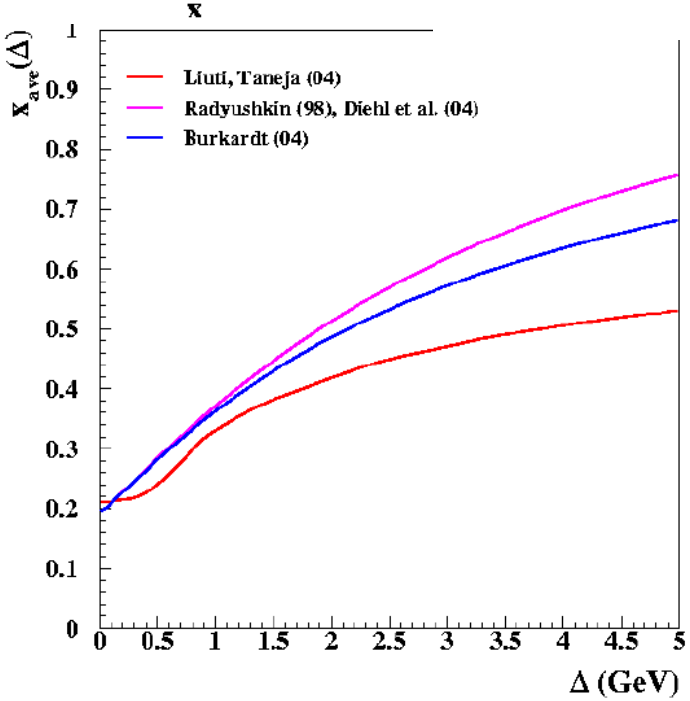
$$q(x,\mathbf{b},\mathbf{b}) = \mid \Psi(x,\mathbf{b}) \mid^2 \equiv q(x,\mathbf{b}), \tag{15b}$$

$\langle k_T^2 \rangle$ vs. x



$\langle r^2 \rangle = \langle b^2 \rangle / (1-x)$ vs. x *(M. Burkardt)*





$\langle x \rangle$ vs. Δ

S.L. and S.K. Taneja, (2004)

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Conclusions

✓ We uncovered a non-trivial partonic interpretation of GPDs
FSI important $\rightarrow$ underlying connection with TMDs

✓Dispersion relations are not directly applicable: all information is not on the “ridge”. All measurement (real and imaginary parts) are important.

✓ Connection between GPDs and TMDs embedded in k_T dependent quantities (SL and Taneja, 2004)